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develop state space models for aeroelastic systems including unsteady aerodynamics. establish the applicability of semigroup theory for these systems on weighted product aces by showing well-posedness and obtaining a dissipative estimate for the finitesimal generator of the associated Cauchy problem. Numerical techniques are veloped for the singular neutral functional differential equation that model the roelastic systems. Also, we present a technique based on quasilinearization for entifying unknown coefficients in parabolic partial differential equations. This chnique is applied to both linear and nonlinear equations with spatially varying known coefficients.

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State Space Models for Aeroelastic and

Viscoelastic Systems

Final Report: AFOSR-88-0074

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CONTENTS

I. Introduction.

The research described below was completed during the period January 1, 1988, to December 31, 1990. This work was funded by the Air Force of Scientific Research under grant AFOSR-88-0074.

In recent years the feasibility and advantages of active control surfaces to reduce maneuver, gust, and fatigue loads, and dampen vibration that contributes to flutter have been extensively studied. A systematic procedure for control design requires the development of a realistic mathematical model that predicts the dynamical behavior of the physical system. The development of state space models for aeroelastic systems, including unsteady aerodynamics, is potentially import for the design and development of highly maneuverable aircraft. A goal of this research effort was to develop a complete dynamical model for an aeroelastic system in which the elastic motions of the structure are coupled with the motion of the surrounding fluid. Our study included the task of identifying the appropriate state spaces for which the corresponding mathematical model was well-posed as well as the development and analysis of computational algorithms for these systems.

It has been shown that fractional order operators can provide accurate models for viscoelasticially damped structures. The governing equation for such systems has the form of a partial-integral equation. These systems can be viewed as neutral functional differential equations having a structure common to the mathematical models described above for aeroelastic systems. For our study of viscoelastic systems we focused our attention to the task of developing a parameter identification scheme for these systems. As a first step, we investigated the effectiveness of quasilinearization for parameter identification in both linear and nonlinear parabolic partial differential equations. Our study included the heat equation where the unknown parameter was the spatially varying diffusion coefficient and the nonlinear Burgers' equation.

II. Aeroelastic Systems.

A completed dynamic model for the elastic motions of a three–degree–of–freedom "typical" airfoil section, with flap, in a two–dimensional incompressible flow was formulated by Burns, Cliff and Herdman, (see [1, reference 8]), in terms of a functional differential equation of neutral–type. In subsequent papers, Burns, Herdman and Turi (see [1, references 10–13 and 24]), the well–posedness of the modeling neutral equation was studied in a product space framework. The analysis showed that the dynamic model extends to a well–posed state–space model on the product space $\mathbb{R}^8 \times L_p$; $p \geq 1$ if and only if $p < 2$. Since the ultimate goal of the modeling process was to generate a framework for the design of active control schemes for flutter suppression these results would suggest that one consider histories belonging to L_p, $p < 2$. On the other hand in the derivation of the evolution equation for the circulation on the airfoil one has to assume that the circulation history belongs to L_p for $p > 2$ in order to guarantee the applicability of Söhngen's inversion formula for obtaining a representation for the solution of the airfoil equation. Motivated by these observations, we investigated the problem of finding a state–space such that Söhngen's inversion formula was applicable and at the same time the resulting neutral system was well–posed. That is, we wanted to identify conditions on the initial data, including the past history, to guarantee well–posedness of the model equations that matched the conditions necessary to justify the validity of the inversion formula employed to obtain the solution of the airfoil equation. During this time period we have shown that this compatibility can be obtained by assuming that the circulation history belongs to a weighted L_2 space.

Our study of the well–posedness of functional differential equations, including the aeroelastic system discussed above, has been in the context of functional analytic semigroup theory. The applicability of the semigroup approach for the development of approximation techniques for parameter estimation and optimal control for such system requires the identification of an appropriate state–space which not only yields the C_0–semigroup but also allows one to obtain a dissipative estimate for the infinitesimal generator for the semigroup.

This requirement is due to the convergence of the numerical schemes being established by using the Trotter–Kato semigroup approximation theorem. This dissipative estimate for the aeroelastic system with state–space $\mathbb{R}^8 \times L_2$ is guaranteed, however, finding the equivalent norm on that space, which is needed to give the dissipative estimate, is not straightforward. For retarded functional differential equations and atomic neutral functional differential equations on the state space $\mathbb{R}^n \times L_{2,g}$, g a weight function, a well–developed theory is available. However, for singular neutral systems, such as our aeroelastic system, a characterization of g that is needed to obtain the dissipative estimate was unknown. Our investigation shows that the selection of g plays a key role in the development of the semigroup formulation for our system.

In [2] we establish that the finite delay version of the aeroelastic system generates a C_0–semigroup on the product space $\mathbb{R}^7 \times L_{2,g}$ and show that the infinitesimal generator of that semigroup does indeed satisfy a dissipative estimate. Here the function g is defined by $g(s) = [1 + (-s)^{-1}]^{\frac{1}{2}}$. Our framework can be extended to include more general singular neutral functional differential equations. In the general case we allow a weight function g satisfying (i) $g(s) > 0$, $g(\cdot) \in L_1$, $\dot{g} \geq 0$ and (ii) for any given $c > 0$, there is a $\tau > 0$ such that $\dot{g}(s) \geq cg(s)$ on $[-\tau, 0]$. In [3] we present an approximation scheme and provide two illustrative examples to indicate the feasibility of our approach. One example is a scalar equation with inconsistent initial data, no smoothness on the solution at $t = 0$, the other example is a two–dimensional system which exhibits characteristics similar to those one observes in the aeroelastic system (see [2,3]).

In [1] we study the derivation of the equations for the aeroelastic system assuming that the circulation history belongs to a weighted L_2 space. We establish that the assumption that the "past history" of the derivation of the total circulation function belongs to $L_{2,g}$, same g that provided the dissipative estimate, is sufficient to assure to validity of the Söhngen's inversion formula for the solution of the airfoil equation. Therefore, we have obtained compatibility of the validity of the inversion formula and the well–posedness of the resulting neutral system. The resulting state space $\mathbb{R}^7 \times L_{2,g}$ and the numerical techniques described above provides a suitable setting for control design for the aeroelastic system.

For the complete system described above the evolution equation for the circulation on the airfoil was coupled to the rigid–body dynamics of the airfoil to obtain the infinite delay singular neutral system of the form

$$\frac{d}{dt}\left[Ax(t) + \int_{-\infty}^{0} A(s) \times (t+s)\, ds\right] = Bx(t) + \int_{-\infty}^{0} B(s) \times (t+s)\, ds + f(t) \tag{2.1}$$

as the mathematical model. In (2.1) the 8×8 matrix A is singular, the 8×8 matrix function $A(s)$ is weakly singular, $A_{88}(s) = ((Us-2)/Us)^{\frac{1}{2}}$ and f represents a forcing term (possible control) for the system. The state of the system (2.1) includes the past history of $\dot{\Gamma}$ where Γ denotes the total airfoil circulation (see [1]). This past history of $\dot{\Gamma}$ may not be observable for the entire past time $(-\infty, 0)$. However, one would be able to observe this past history over a finite time interval say $[-r, 0]$. Since the kernel function $A(s)$ is not integrable on $(-\infty, 0)$ it is not possible to address the infinite delay problem as a finite delay problem by *mapping the interval* $(-\infty, 0)$ *to some finite interval* $[-T, 0]$. On the other hand the "finite delay" version of (2.1)

$$\frac{d}{dt}\left[Ax(t) + \int_{-r}^{0} A(s) \times (t+s)\, ds\right] = Bx(t) + \int_{-r}^{0} B(s) \times (t+s)\, ds + f(t) \tag{2.2}$$

as mention earlier has been extensively studied. During this period we have investigated the possibility of taking advantage of our results for the finite delay system to study the infinite delay system. In [4] we develop an approximation technique for (2.1) on an interval of the form $[0, T]$ using the approximation techniques we developed for the finite delay system (2.2). In order to accomplish this we have made use of the special structure found in the aeroelastic system. In particular, the representation of the aeroelastic system allows one to view the system as having an integrodifferential Volterra component and a singular integral component (see [4]). We have shown that the infinite delay singular integral component found in (2.1) can be approximated using the corresponding finite delay singular integral component of (2.2). Our results including error estimates for the approximation are presented in [4].

Our two step approximation procedure starts with truncating the infinite delay term. The approximation is dependent on the specified initial data. The desired error bound together with the time interval $[0, T]$ on which one wishes to find the solution dictates the necessary truncation. The second step consists of employing the approximation scheme established for the associate finite delay system.

III. Parameter Identification.

During this period we developed a scheme for identification of unknown parameters in parabolic partial differential equations. The task that we undertook was to develop a quasilinearization based algorithm that could be used to identify parameter for nonlinear as well as linear parabolic partial differential equations. As a first step we considered a parameter identification for the heat equation where the unknown parameter was the spatially varying diffusion coefficient. Our study included theoretical detail and careful numerical testing. We established the Fréchet differentiability of the solution with respect to the unknown parameter which was shown to be a primary result necessary to ensure local convergence of the approximation scheme used to find the unknown parameter. The numerical examples that we present demonstrate the rapid convergence and accuracy of the algorithm. Our efforts to date involving the use of quasilinearization in nonlinear partial differential equations have only been numerical ones. However, the success of our numerical testing leads us to believe that our quasilinearization scheme will provide an effective approach to parameter identification for certain nonlinear equations. Our results are presented in [5].

IV. Interaction with Air Force Laboratories and Industry.

During the period of this grant, Dr. Herdman has consulted with Air Force Laboratory and Industry personnel for motivation and guidance for this research effort. We discussed our research results and possible implementation with the following people:

- Wen–Huei Jou, Manager, CFD Development, Computational Fluid Dynamics Lab., Boeing Commercial Airplanes, Seattle, Washington.
- Lewis F. Jurey, Manager, Flutter and Dynamics, Advanced Development Projects, Lockheed Aeronautical Systems Company, Burbank, California.
- J. Jenkins and J. Lee, Wright Research and Development Center, Wright–Patterson AFB, Ohio.

V. Personnel Supported.

Principal Investigator: Terry L. Herdman–Professor, Department of Mathematics.

Graduate Project Assistant: Patricia W. Hammer–Ph.D. Student, Department of Mathematics.

VI. Publications.

The research funded by the grant has produced the following papers:

[1] T. L. Herdman and J. Turi, An application of finite Hilbert transforms in the derivation of a state space model for an aeroelastic system, Journal of Integral Equations and Applications, Vol. 3, Number 2, Spring 1991, to appear.

[2] T. L. Herdman and J. Turi, Singular integral equations, New Trends and Applications of Distributed Parameter Control Systems, Lecture Notes in Pure and Applied Mathematics, Vol. 128, G. Chen, E. B. Lee, W. Littman and L. Markus, Eds., Marcel Dekker Inc., 1990.

[3] T. L. Herdman and J. Turi, On the solutions of a class of integral equations arising in unsteady aerodynamics, Differential Equations: Stability and Control, Lecture Notes in Pure and Applied Mathematics, Vol. 127, S. Elaydi, Ed., Marcel Dekker Inc., 1990, pp. 241–248.

[4] E. M. Cliff, T. L. Herdman and J. Turi, Finite Memory Approximations for a singular neutral system arising in aeroelasticity, 30th IEEE Conference on Decision and Control, submitted.

[5] P. W. Hammer, Parameter identification in parabolic partial differential equations using quasilinearization, Ph.D. thesis, VPI&SU, Blacksburg, Virginia, 1990, ICAM Report 90-07-01.